Polarization Control in Terahertz Metasurfaces with the Lowest Order Rotational Symmetry

Longqing Cong, Ningning Xu, Weili Zhang, and Ranjan Singh*

Symmetry is fundamental to the universe and the laws that govern the nature. Symmetry has been studied and explored in different disciplines, such as biology, physics, mathematics, music, and arts. In mathematics, symmetry is more precisely defined as a property that is invariant to a transformation with respect to the passage of time. [1,2] The study of symmetry breaking induced changes in chemistry and biology is particularly important as molecules with different spatial configurations result in significantly different responses. [3–5] Recently, parity-time (PT) symmetry breaking in non-Hermitian wave equations has been intensely discussed by balancing the gain and loss in optics that has been applied in microring lasing. [6–9]

The atoms in a solid are locked in a crystalline lattice that leads to different vibrational frequencies, and hence it has different refractive indices in different directions. In general, the polarization of electromagnetic waves is governed by the real part of the susceptibility tensor. However, to arbitrarily control the spatial or planar arrangement of natural materials at the atomic scale is not very straightforward. Recently discovered artificially designed metasurfaces with subwavelength building block unit cells allow excellent control on the optical properties of the effective medium. [10–15] These engineered metasurfaces have been utilized to design photonic devices with exotic properties such as controlling light propagation, [12] perfect lenses, [11] sensing and imaging devices, [16,17] and invisibility cloaking. [13,18] Besides the manipulation of free space light, metasurfaces have also revealed the ability to enhance the polarization conversion and coupling of free space wave to surface plasmons. [19] With the concept of metasurfaces, [20,21] we can design metamolecules with specific symmetry to control the polarization state of the incoming light. Owing to the 2D feature of metasurfaces, we could degenerate the tensor \( \chi \) from complex 3 × 3 matrix to 2 × 2 matrix. [10] The transmission response through a metasurface can be described by transmittance, polarization ellipses, averaged polarization rotation, and polarization conversion, [22–25] which can easily be determined from the frequency-dependent 2 × 2 Jones matrix. [26,27]

In the recent past, there have been several works on chiral metamolecules that consist of 2D and 3D designs [28,29]. For the 2D metasurfaces with planar chirality, the most commonly employed designs have fourfold rotational symmetry. [30–33] Asymmetric transmission of circular polarization in planar metamaterial structures has been discussed in the past. [34–39] Another approach to obtain the optical activity by planar metasurfaces is to tilt the metamaterial plane with respect to the incident light so that the excited magnetic and electric dipoles in the asymmetric metamolecule interact with each other. [40] However, the optical activity and birefringence are not strong enough in these metamaterials to have a functionality such as a circular polarizer and polarization rotator. Therefore, 3D designs and multilayer metamaterials were proposed to improve the optical activity and birefringence. [31,41,42] Circular polarizers with broadband operation were also realized due to the giant circular dichroism of the 3D and multilayer designs. A major advantage of 3D chiral metamolecules over planar chiral design is that the asymmetric transmission of linearly polarized light can be achieved by breaking the symmetry of the unit cell in the propagation direction. [43–45] However, the complex fabrication process limits the application of these devices to a large extent. Therefore, a simple planar design with giant optical activity and asymmetric transmission of circularly polarized light becomes highly desirable.

Unlike the previously employed designs, we propose a planar metamolecule with the lowest order rotational symmetry, i.e., twofold rotational symmetry, and demonstrate a giant optical activity and asymmetric transmission of the terahertz waves. We observed a giant cross polarization transmission response, which is the most straightforward way to determine the optical activity, the polarization control as well as realize high efficiency wave deflection using planar metasurfaces. The major advantage of the twofold rotational symmetric planar structure is due to the additional degree of freedom that allows us to tailor the optical axes of the metamolecule that can in turn modulate the cross polarization transmission amplitude. By varying the orientation of the split gaps in the split-ring resonator (SRR) metamolecule and preserving the rotational symmetry, the intrinsic optical activity and asymmetric transmission is easily modulated. The optimized design allows the strongest optical activity and an excellent bidirectional circular polarizer functionality.
Our experimental design is based on SRR unit cell that consists of two gaps placed at the center of top and bottom arms possessing dual (x and y) axis symmetries (AS) and twofold rotational symmetry (RS), as shown in Figure 1a, named as s-SRR. We study the relationship between optical activity and structural symmetry based on the chosen metamolecule. By altering the position of the split gaps, the symmetry of the structure could be altered. When these two gaps are displaced by equal distance \( \delta x \) in the opposite directions as shown in Figure 1b, the SRR structure retains the twofold rotational symmetry, i.e., C2 rotational symmetry, while the x and y axis symmetries are broken. Therefore we would address this set of structures as rotationally symmetric SRRs, r-SRRs. When these two gaps are displaced equally by distance \( \delta x \) in the same direction as shown in Figure 1c, the x-axis symmetry is preserved but the y-axis and the C2 rotational symmetries are broken. We would address this set of structures as x-axis symmetric SRRs, x-SRR.

For the periodically arranged subwavelength unit cells, we could assume that the fabricated structures are symmetrically embedded with all the materials being linear and reciprocal. The metasurfaces are illuminated by a plane wave propagating in the positive z direction \( E_z(t) = \left( \begin{array}{c} i_x \\ i_y \end{array} \right) e^{i(\omega t - kz)} \) with \( \omega \) being the frequency, \( k = \omega / c \sqrt{\varepsilon(\omega)} \) being the wave vector, and the complex amplitudes \( i_x \) and \( i_y \) describing the states of polarization. The transmitted field is then written as 

\[
E_i(\vec{r},t) = \left( \begin{array}{c} t_x \\ t_y \end{array} \right) e^{i(\omega t - kz)},
\]

where \( \varepsilon(\omega) \) is the permittivity of the dielectric medium. With coherent plane waves, a generalized Jones calculus (T matrix) could be employed instead of the Mueller calculus necessary for incoherent light.\([7,16]\) The relation between incident and transmitted field based on the T matrix is given by 

\[
\begin{pmatrix} t_x \\ t_y \end{pmatrix} = \frac{1}{\varepsilon} \begin{pmatrix} T_{xx} & T_{xy} \\ T_{yx} & T_{yy} \end{pmatrix} \begin{pmatrix} i_x \\ i_y \end{pmatrix} = \hat{T} f \begin{pmatrix} i_x \\ i_y \end{pmatrix},
\]

where the superscript \( f \) designates the propagation in the forward direction. By taking symmetry into account, we perform the matrix operation while rotating the metasurfaces by arbitrary angles \( \varphi \) with respect to the z-axis:

\[
D_x = \begin{pmatrix} \cos(\varphi) & \sin(\varphi) \\ -\sin(\varphi) & \cos(\varphi) \end{pmatrix},
\]

then the new T matrix \( \hat{T}_{\text{new}} \) is transformed by \( \hat{T}_{\text{new}} = D_x^{-1} \hat{T} D_x \). Therefore, for our designed r-SRR with C2 symmetry, i.e., \( \varphi = 180^\circ \), \( D_{180} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \), and

\[
\hat{T}_{\text{new}} = D^{-1}_{180} \hat{T} D_{180} = \begin{pmatrix} T_{xx} & T_{xy} \\ T_{yx} & T_{yy} \end{pmatrix} = \hat{T},
\]

which signifies that there would not be any specific feature of T matrix that would appear with mere symmetry breaking along the x and y axes in s-SRR. Similarly, the operator is given by \( D_x = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \) when the reflection of the unit cell is considered relative to the x-axis. Thus, we obtain the transformation T matrix of the mirror structure as \( \hat{T}_{\text{new}} = D_x^{-1} \hat{T} D_x \). The T matrix of the reflected counterpart should be identical to the original one for r-SRR with x-axis symmetry, which gives rise to 

\[
\hat{T}_{\text{new}} = \begin{pmatrix} T_{xx} & -T_{xy} \\ -T_{yx} & T_{yy} \end{pmatrix} = \hat{T}^f = \begin{pmatrix} T_{xx} & T_{xy} \\ T_{yx} & T_{yy} \end{pmatrix} = \hat{T} = \begin{pmatrix} T_{xx} & 0 \\ 0 & T_{yy} \end{pmatrix}.
\]

Such a diagonal T matrix without \( T_{xy} \) and \( T_{yx} \) can explicitly describe the response of x-SRR and also highlight the inability to obtain optical activity by using planar metasurface that possesses any axis of symmetry. Therefore, the existence of the cross-polarized transmission coefficients \( T_{xy} \) and \( T_{yx} \) should be the differentiating factor between x-SRR and r-SRR that also determines the ability of r-SRR to achieve high optical activity.

We fabricated the metasurfaces using standard photolithography technique and the designs have been displayed in Figure 1a–c. Three representative sets of samples exhibit how the relationship between optical activity and structural dimensions of the two gap SRRs are kept constant. In order to probe the influence of symmetry breaking and demonstrate the theory above, we fabricated s-SRR sample with \( \delta x = 0 \) and r-SRR and x-SRR samples with \( \delta x = 10 \mu m \) and \( 20 \mu m \), respectively. All the other structural dimensions of the two gap SRRs are kept constant. We explored the cross-polarized electric field conversion in metasurface samples with different types and degrees of symmetry in experiments. Finite-element frequency-domain (FEFD) solver with unit cell boundary condition was applied for the unit cell shown in Figure 1d. The silicon substrate was modeled as a lossless dielectric with \( \varepsilon = 11.78 \) and metal aluminum has a conductivity of \( \sigma = 3.72 \times 10^7 \) S m\(^{-1}\). We chose this metal due to the
high electrical conductivity at the terahertz frequencies and the ease of handling the metasurface samples, which does not get oxidized easily when exposed to atmosphere while performing the measurements. The simulated cross-polarized transmission spectra has been shown in Figure 2b. The response of symmetry broken SRRs was also experimentally demonstrated through measurements using the 8-f confocal terahertz time-domain spectroscopy (THz-TDS) system with three wire-grid polarizers.\(^{[47–49]}\) The detailed experimental setup has been discussed in the experimental section. In experiments, the cross-polarized radiation was measured after the transmission of terahertz waves through the samples with the \(y\)-direction linearly polarized (\(s\)-polarized) incidence. The amplitude transmission coefficient of \(x\)-polarized (\(p\)-polarized) electric field, i.e., \(|T_{xy}|\), is shown in Figure 2c for all the five different metamaterial samples. As we could observe, the simulated and measured results have excellent agreement. The different transmission response is distinctly exhibited by comparing \(|T_{xy}|\) spectra between \(r\)-SRRs and \(x\)-SRRs. In \(x\)-SRRs, extremely weak cross-polarized electric fields are induced that mainly originates from the noise in the experiment. However, we observe a strong cross-polarized component with the highest transmission amplitude of 0.47 for the case of \(\delta x = 20 \mu m\) in \(r\)-SRRs. Our measurements show that we nearly achieved the theoretical limit 0.5 of the cross polarization amplitude conversion by a planar metasurface design that has also been discussed recently by Ding et al.\(^{[50,51]}\) A similar response occurs for cross-polarization transmission with \(x\)-polarized incident light, i.e., \(|T_{yx}|\) (see the Supporting Information). We then demonstrate the Jones calculus predictions according to the intrinsic symmetry breaking in SRRs. More interestingly, the transmission amplitude of the cross-polarized light in \(r\)-SRRs could be tuned from 0.31 to 0.47 by varying the \(\delta x\) parameter from \(\delta x = 10 \mu m\) to \(\delta x = 20 \mu m\), which endows the twofold rotationally symmetric design with an alternative degree of freedom to modulate the optical activity. This rotational symmetry induced cross-polarization conversion is a key ingredient to induce optical activity and design both active and passive polarization control devices using flat metasurfaces.

In order to elucidate the underlying mechanism of the observed spectral characteristics of \(r\)-SRRs and obtain a numerical relation, we analyze the evolution of surface electric fields distribution in both \(x\) and \(y\) directions at the electric dipole resonance frequency of 0.7 THz with \(y\)-polarized incidence, as shown in Figure 3a,b. The electric fields polarized along the \(y\)-direction are distributed merely in the top and bottom arms in all the three samples as shown in Figure 3a. With increasing \(\delta x\) parameter, the intensities of electric dipoles become weaker, which suggests weaker resonance of the \(y\)-polarized light from the metasurface. When looking at the \(x\)-polarized \(E\)-field distribution, the dipoles appear on the left and the right vertical arms of the SRRs, as well as at the split gaps as shown in Figure 3b. As the axial symmetry is broken, the axis symmetry of dipole distribution of \(x\)-polarized field is also broken, which, therefore, induces the cross-polarized components radiating in the \(x\)-direction. However, contrary to the distribution of \(y\)-polarized components, the intensities of the \(x\)-polarized fields become
stronger with increase in $\delta x$, indicating that a large fraction of incident energy of the $y$-polarized light gets converted into the $x$-polarized radiation after transmitting through the rotationally symmetric SRRs. The E-field energy density distribution demonstrates the energy conversion from the $y$-polarized light to the $x$-polarized radiation as shown in Figure 3c, which clearly shows the transition in the direction of the total radiated energy from the top and bottom horizontal arms of the $r$-SRRs to the left and right sides of the vertical arms.

According to the above analysis, we further investigate the numerical relationship between the gap displacement $\delta x$ and the cross-polarized transmission amplitude. Here, we define the effective rotation degree $\alpha_{\text{eff}}$ for a C2 rotational symmetry configuration as shown in Figure 3d. The relationship between $\delta x$ and $\alpha_{\text{eff}}$ can be described as $\sin \alpha_{\text{eff}} = \frac{\delta x}{\sqrt{\delta x^2 + l_{\text{eff}}^2}}$ where $l_{\text{eff}}$ refers to the effective side arm length of the $r$-SRR. $\alpha_{\text{eff}}$ is zero for $s$-SRR due to the existence of mirror symmetry. As discussed above, the cross-polarized amplitude gets enhanced with the increase in $\delta x$. We further confirm this by plotting the cross-polarized amplitude for $r$-SRR by varying $\delta x$ from $\delta x = -20$ (anticlockwise rotational symmetry) to $+20$ μm (clockwise rotational symmetry) in steps of 2.5 μm at 0.7 THz, as shown in Figure 3e. To give the numerical relationship between $\alpha_{\text{eff}}$ and $|T_{xy}|$ at this frequency, we assume the simplest case that $|T_{xy}|$ is proportional to the sinusoidal function of $\alpha_{\text{eff}}$, i.e., $|T_{xy}| \propto \sin \alpha_{\text{eff}}$.

Then, we fit the function $|T_{xy}| = k \frac{\delta x}{\sqrt{\delta x^2 + l_{\text{eff}}^2}} \quad (-20 \leq \delta x \leq 20)$ with $l_{\text{eff}} = 13.776$ μm and $k = 0.588$, where $k$ is the proportionality constant. From the fitting curve, it matches well with the simulated data and explicitly describes the numerical dependence of the cross-polarized transmission amplitude on $\delta x$. In fact, the twofold rotational symmetry is not a mandatory condition to induce the cross polarized transmission. However, the highest conversion efficiency occurs when the unit cell possesses a twofold rotational symmetry (see the Supporting information).

Thus far, we have clarified the mechanism of $r$-SRR to induce the cross-polarized component and demonstrated an extra degree of freedom to modulate this component. Next, we will discuss the asymmetric transmission and optical activity of $r$-SRR due to the cross-polarized component.

For a 2D planar metasurface, the Jones matrix should be given by $\hat{T}_r = \begin{pmatrix} T_{xx} & -T_{xy} \\ -T_{yx} & T_{yy} \end{pmatrix}$ for light propagating along the backward direction in a fixed coordinate system by applying the reciprocity theorem of four-port systems in the absence of an external magnetic field. The Jones matrix for backward transmission coincides with the mirror image when observed from the front side of $r$-SRR, which is $\hat{T}_r = \begin{pmatrix} T_{xx} & T_{xy} \\ T_{yx} & T_{yy} \end{pmatrix}$, and therefore we obtain $T_{px} = T_{xy}$, which means that the two cross-polarized transmission with $x$-polarized and $y$-polarized incidence would possess the same behavior including the amplitude and phase information. Thus, there would not be any asymmetric transmission for the linearly polarized light according to $\Delta_{\text{in}} = -\Delta_{\text{in}} = |T_{px}|^2 - |T_{xy}|^2 = 0$ as demonstrated by the experiments and analysis, where $\Delta = |T_{px}|^2 - |T_{xy}|^2$ represents the asymmetric transmission of the linearly polarized light. However, the asymmetric transmission of circularly polarized light occurs when the metasurface is excited by a linearly polarized light. To be clearer, when a linearly polarized light is incident from the
forward direction, the output polarization state of the light is right-handed circular polarization and when the same linearly polarized light is incident from the rear side of the metasurface, the output state is left-handed circular polarization.

Due to the axis symmetry breaking, the rotational symmetry introduces planar chirality in r-SRR. For intrinsic chirality, the induced magnetic (electric) dipoles should be parallel to the excited electric (magnetic) field. In this case, the eigen polarizations correspond to the circular polarization of light, whereas they are elliptic in the more general nonparallel case, i.e., bianisotropic. It is next to impossible to realize intrinsic chirality with the planar metasurfaces. However, we still obtain excellent circular polarization output state by designing a simple planar metasurface with the lowest order rotational symmetry, which is based only on two oscillating electric dipoles. Circular dichroism (CD) and optical rotation are usually the measure of the optical activity in chiral materials, which mainly refer to the difference between transmission spectra of right-handed (RH) and left-handed (LH) circularly polarized light, and therefore is related to the ellipticity of the transmitted wave. We estimated CD as \( \theta = \tan^{-1}\left(\frac{T_R - T_L}{T_R + T_L}\right) \) and optical rotation as \( \phi = \arg(T_R) - \arg(T_L) \), where \( T_{RL} \) is transmission signal for RH (LH) circularly polarized light (see the Experimental Section). For a simple planar metasurface, \( T_{RR} = T_{LL} \) since \( T_{xy} = T_{yx} \), and therefore, all the planar circular dichroism is due to \( T_{LR} \neq T_{RL} \).

As shown in the shaded area of Figure 4a–c, with the increase in rotation degree by tuning \( \delta x \) from \( \delta x = 0 \) to \( \delta x = 20 \) \( \mu \)m, the amplitude transmission of RH and LH circularly polarized light shows an increasing difference. The CD and optical rotation spectra explicitly demonstrate the tuning of the optical activity in r-SRR by varying the rotation degree of the SRR split gap locations, as shown in Figure 4d,e. The largest CD and rotation at 0.9 THZ reach 40.7° (44.8° in simulation) and 53° for \( \delta x = 20 \) \( \mu \)m, which are much larger compared to previous observations in the 3D chiral structure and prove to be an excellent candidate as a flat circular polarizer. The difference between \( T_{LR} \) and \( T_{RL} \) also induces the asymmetric transmission of circular polarized light when we compare the incident linearly polarized light from the forward and backward directions at normal incidence. This asymmetric transmission property also induces the switching of the optical activity and polarization conversion property of the metasurface device when observed from the forward and the backward directions, respectively.

In Figure 5a, we exhibit the measured co-polarized amplitude transmission coefficient \( |T_{yy}| \), which reveals the obvious amplitude modulation. Normalized ellipticity is calculated using Stokes parameters (see the Experimental Section) and shown in Figure 5b. The output light is the linearly co-polarized state for s-SRR (\( \delta x = 0 \)) and as the axial symmetry is broken for r-SRR, RH elliptically and circularly polarized light appears at around 0.9 THz with high transmission power as shown in Figure 5e. In addition, we observe the broadening in the bandwidth with the increase in the rotational degree to the appearance of another eigen resonance of the SRR structure.

![Figure 4](https://www.advopticalmat.de/figs/0000000404.png)

**Figure 4.** Optical activity in twofold rotationally symmetric SRR. The measured output transmission amplitude for right-handed and left-handed circular polarization when a) \( \delta x = 0 \) \( \mu \)m, b) \( \delta x = 10 \) \( \mu \)m, and c) \( \delta x = 20 \) \( \mu \)m. d) The measured circular dichroism, \( \theta \), and e) optical rotation, \( \phi \) spectra for the three samples.
Since the optical axes are no longer at $x$ and $y$ axes while the axis symmetry of $s$-SRR is broken, the eigen resonance mode related to the $x$-polarization can be excited. The detailed explanation can be found in the Supporting Information. We also show the corresponding simulated data in Figure 5c,d that is in excellent agreement with the measurements.

By probing the optical activity of the structure, the rotation of $r$-SRR is clockwise when $\delta x$ is positive, which transmits RH circularly polarized light and attenuates the opposite circular polarization state.\(^\text{[41]}\) When looking at the sample from the rear side, $r$-SRR exhibits anticlockwise rotation where $\delta x$ is negative and shows as the mirror counterpart of the original $r$-SRR relative to $x$ or $y$-axis, as shown in the inset of Figure 5f. Under such a condition, $r$-SRR transmits LH circularly polarized light as shown in Figure 5f by the calculated ellipticities, which appear consistent with the optical activity analysis. From a practical application point of view, such a planar circular polarizer can function as an RH circular polarizer when light transmits from the front side and an LH circular polarizer when light transmits from the back side. Although the operation bandwidth is narrower than the 3D helix metamaterial circular polarizer,\(^\text{[41]}\) planar $r$-SRR exhibits the asymmetric transmission when the light is incident either from the front or from the rear surface.\(^\text{[35]}\) The operation bandwidth can be broadened by stacking several layers of metasurfaces.\(^\text{[22]}\)

In conclusion, we have discovered the cross polarization transmission response of a flat terahertz metamaterial with the lowest order rotational symmetry. The ultrahigh efficiency of cross polarization transmission would enrich the design of wave deflection device by carefully engineering the phase discontinuities in metasurfaces. Giant optical activity and asymmetric transmission of circularly polarized light have been observed. The twofold rotational symmetric structure offers an extra degree of freedom to modulate the optical activity and asymmetric transmission by tailoring the optical axis of the planar metamolecule. This flexible design would open up avenues for applying flat metasurfaces to terahertz optics. It would also enable the design and development of active polarization conversion devices by integrating novel dynamic material in the planar configuration to have functionalities such as polarization sensitive detection, polarization filters, and chiral active terahertz sensing.

**Experimental Section**

**Measurements:** At normal incidence, the metamaterial samples were characterized by the antenna-based THz–TDS system with three sets of wire-grid polarizers P1, P2, and P3 as shown in Figure 6.\(^\text{[48,49]}\) The near-infrared laser beam (50 fs, 6 nJ per pulse at 800 nm with 80 MHz repetition rate) was applied to generate and detect terahertz signal. Since the antennas were fixed in THz–TDS, and the emitter and receiver polarization was fixed, the sample had to be rotated to obtain orthogonal polarized incident signal. However, the coordinate will be rotated at the same time by rotating samples, which will introduce additional 180° phase difference between $T_{xy}$ and $T_{yx}$, and this would be eliminated during the data process. To obtain the cross transmission signal, three sets of polarizers were inserted into the terahertz path between the sample and the receiving antenna with P1 right after the sample to filter the co-polarized signal and P2 and P3 arranged to rotate the transmitting cross-polarized signals from P1 to the co-polarized direction, which could then be detected by receiving antenna. For co-polarized transmission of samples and corresponding reference (blank Si) signals, P1 was simply rotated by 90° and these were measured again. The transmission coefficients were obtained by $|T_{ij}(\omega)| = |E_{ji}^r(\omega)/E_{ij}^l(\omega)|$ and the corresponding phase information was extracted by $\phi_{ij} = \arg(E_{ji}^r(\omega)/E_{ij}^l(\omega))$ where $E_{ji}^r(\omega)$ and $E_{ij}^l(\omega)$ were fast Fourier transformed transmitted electric fields of sample signals.
and reference pulses, and \(i\) and \(j\) represented incident and output polarization components.

Calculations: To obtain the transmission coefficients of circularly polarized waves, that is, \(T_{RR}, T_{RL}, T_{LR},\) and \(T_{LL}\), all the four linearly co-polarized and cross-polarized components needed to be measured, including \(T_{xx}, T_{xy}, T_{yx},\) and \(T_{yy}\). With vectors \(\mathbf{T}\) and \(\overline{\mathbf{T}}\) denoting the incident and transmitted light in a certain base, the light in the Cartesian base was written as \(i = \mathbf{A} \mathbf{T}\) and \(\mathbf{t} = \mathbf{\overline{A}} \mathbf{\overline{T}}\), respectively, with \(\mathbf{A}\) being the operator of the basis matrix. Hence, the new \(T\) matrix of the new base was given by \(T = \mathbf{\Lambda} \mathbf{^T A} = \begin{pmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{pmatrix}\). For a circular base, the new operator was \(\mathbf{\Lambda} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix}\), and therefore transmission coefficients of circularly polarized light in a circular base\(^{[55]}\) could be described based on the linearly polarized states coefficient in the Cartesian base as

\[
\mathbf{t}_c = \begin{pmatrix} T_{RR} & T_{RL} \\ T_{LR} & T_{LL} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} (T_{xx} + T_{yy}) + i(T_{xy} - T_{yx}) & (T_{xx} - T_{yy}) + i(T_{yx} + T_{xy}) \\ (T_{xx} - T_{yy}) + i(T_{yx} + T_{xy}) & (T_{xx} + T_{yy}) - i(T_{yx} + T_{xy}) \end{pmatrix}
\]

where \(R\) and \(L\) represented right-handed and left-handed circularly polarized light, and \(x\) and \(y\) were orthogonal linearly polarized components oriented in the \(x\) and \(y\) directions, respectively.

Stokes parameters\(^{[55]}\) were employed to calculate the output polarization states with four basic parameters at \(\gamma\)-polarized incidence

\[
\begin{align*}
S_0 &= |T_{x\gamma}|^2 + |T_{y\gamma}|^2 \\
S_1 &= |T_{x\gamma}|^2 - |T_{y\gamma}|^2 \\
S_2 &= 2|T_{x\gamma}||T_{y\gamma}| \cos \varphi_{\text{diff}} \\
S_3 &= 2|T_{x\gamma}||T_{y\gamma}| \sin \varphi_{\text{diff}}
\end{align*}
\]

where the first and second subscripts referred to the transmitted and incident electric field polarization and \(\varphi_{\text{diff}} = \arg(T_{y\gamma}) - \arg(T_{x\gamma})\). The figure of merit of the output polarization states was defined through the normalized ellipticity \(\chi\) as \(\chi = \frac{S_1}{S_0}\), where \(\chi = 1\) indicated a perfect right-handed circularly polarized light, \(\chi = -1\) indicated a perfect left-handed circularly polarized light, and \(\chi = 0\) indicated the perfect linearly polarized light\(^{[23]}\).

Acknowledgements

L.C. and N.X. contributed equally to this work. L.C. and R.S. acknowledge NTU startup Grant No. M4081282 and MoE Tier 1 Grant No. M4011362 for funding of this research. This work was partially supported by the U.S. National Science Foundation (Grant No. ECCS-1232081).

Received: February 13, 2015
Revised: March 26, 2015
Published online: